

Dilute Liquid of Instanton and Its Topological Charge Dominate the QCD Vacuum

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APE smearing and overlap-Dirac operator are combined to filter QCD vacuum configurations. The results obtained from overlap fermions and improved 5Li cooling are compared, both of them exhibit structures of dilute liquid of instanton. Finally the overlap fermions, improved 5Li cooling and APE smearing are combined to calculate the topological charge and identify the structure of QCD vacuum. The results suggest dilute liquid of instanton dominance of topological charge fluctuations in quenched lattice QCD.

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The notion of the gauge field topology is important for understanding QCD vacuum. The instanton liquid model (ILM)[1], where correlation among instantons is introduced, has been used as a basis for developing a rather successful semiclassically-motivated phenomenology. Even though termed as a liquid, it is quite dilute with (anti)instantons of radius $\rho \approx 1/3\text{fm}$ and density $n \approx 1\text{fm}^{-4}$. This setup allows for an interesting mechanism of spontaneous chiral symmetry breaking. Since ILM plays an important role in QCD it has been checked by lattice QCD. It is quite reasonable to expect that the lattice QCD will eventually provide answers about the structure of QCD vacuum. However, finding a clean and satisfactory way to infer this information from lattice QCD has proven to be nontrivial since the Monte-Carlo generated gauge fields fluctuate wildly. In order to eliminate the short-distance fluctuations it is necessary to manipulate the gauge fields by various cooling[2] or smoothing[3] procedures. The cooling is a local minimization procedure for the gauge action with initial one being the Monte-Carlo generated QCD configuration. However, after a few cooling sweeps the gauge field undergoes large changes and become smooth. Further cooling possibly leads to even smoother configuration, and eventually into the trivial configuration with nonperturbative effects removed. Therefore the cooling or smoothing has the subjective nature since one stops at sweeps while the gauge configuration shows the ILM structure.

Horváth et al.[4] used lattice fermions to study the vacuum structure, because the space-time structure of the low-lying eigenmodes of overlap-Dirac operator with the Monte Carlo simulation gauge field is naturally smoother than that of the gauge fields themselves. They identify the possible vacuum structures by finding the local maxima of density $d(n) = \psi_n^\dagger \psi_n$. In their criterion, the local maxima are retained only if the average of $d(n)$ decays monotonically from origin over the distance $\sqrt{3}a$ for all directions[4]. Their results exhibited that the densities of these lump structures increase with the lattice spacing decreasing and might be divergent in the continuous limit. So they concluded that vacuum fluctuations of topological charge are not effectively dominated by instantons. We suspect that the lattice random fluctuations were not excluded in their results. With the lattice spacing decreasing, the random lattice tiny fluctuations become more and more. If they still used $\sqrt{3}a$ in their criterion, which decreases with the decreasing of the lattice spacing a , almost all fluctuations, real and random fluctuations, would be included in their results. Even a real instanton superposed by random fluctuations would be split into small lumps. In order to get rid of this trouble, we combine APE smearing[3, 5, 6], improved 5Li cooling[7, 8] and overlap-Dirac operator[9] methods to fix the space-time structures of the gauge configurations and calculate the topological charge of every structure fixed above to discriminate real and random fluctuations. After the random fluctuations are dropped a dilute instanton liquid QCD vacuum appears.

The APE smearing reduces the short range fluctuations effectively but will not destroy the instantons that are larger than about 1.5 lattice spacing[5]. Furthermore it accelerates the calculation of overlap-Dirac operator approach if one performs pretreatment of the original lattice gauge fields by a few steps of APE smearing. We take $N = 10$ levels of APE smearing with $\alpha = 0.45$ APE parameter for every configuration generated from Iwasaki action[10]. In Fig.1 we give a comparison of the chirality structure (see below) of the lowest nonzero eigenmode of the overlap-Dirac operator with and without pre-treatment by APE smearing. It can be seen from Fig.1 that the structure exhibited by the overlap-Dirac operator with and without pre-treatment are almost the same, but the pre-treated one (Fig.1b) is smoother than that without pre-treatment (Fig.1a). We also compared other low-lying eigenmodes and obtained similar results. These results show that the pre-treatment wouldn't destroy the physical structure and just sweeps away the short range fluctuations.

Our overlap-Dirac operator results show that the density $d(n) = \psi_n^\dagger \psi_n$ and the chirality $c(n) = \psi_n^\dagger \gamma_5 \psi_n$ have the

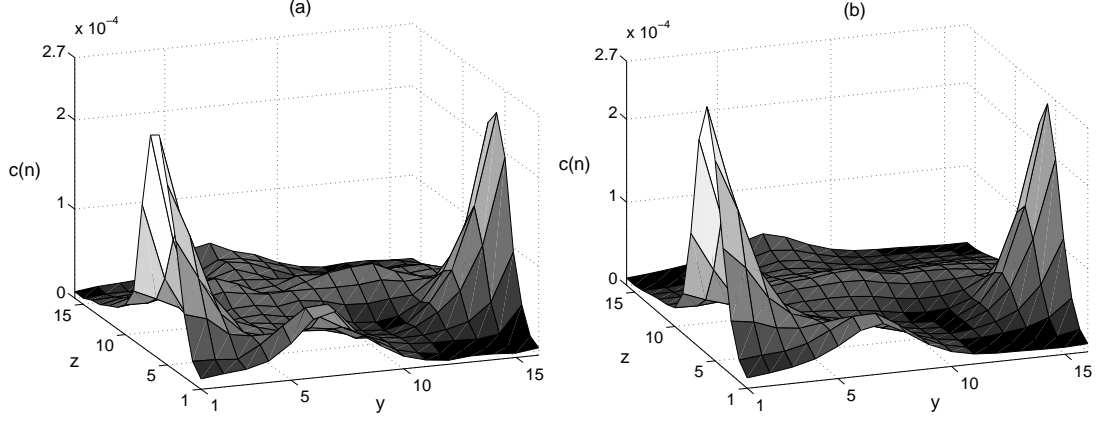


FIG. 1: The distribution of chirality $c(n)$ of the lowest nonzero mode in a y - z slice for a $16^3 \times 20$ Iwasaki lattice ($\beta = 2.746$, $a = 0.08\text{fm}$) without(a) and with(b) APE pre-treatment.

same space-time structures. This is consistent with ILM. However chirality $c(n)$ is better than density $d(n)$, because using of chirality one can distinguish the structures with different chiral property of QCD vacuum. From our overlap-Dirac operator results with APE pre-treated gauge configurations we select out those points which have the biggest chirality and the smallest chirality separately and examine whether they condensate as lumps. Both the zero modes and the lowest nonzero modes of the overlap-Dirac operator are analyzed. As showed in Table 1, different sample of points have been selected to explore the details of structures for a $16^3 \times 20$ lattice. We found that there are a few lumps in the zero mode and the lowest nonzero mode and that when the number of points selected increases some small lumps merge into bigger ones. In the following we will show that the small lumps remained can not be regarded as (anti-)instantons, since topological charge of these structures is far smaller than one; while the bigger lumps have topological charge near the topology integer 1. We also show the distribution of chirality of the lowest nonzero mode in a given time slice in Fig.2(a). This figure exhibits perfect structures of the low-lying mode, which hints that the background gauge field has space-time structures.

TABLE I: The chirality structures of the zero mode and the lowest nonzero mode of a $16^3 \times 20$ lattice at $\beta = 2.746$. The first sub-table lists the structures of the zero mode which is positive chirality. The second one lists the structures of the lowest nonzero mode with positive chirality. The last one lists the structures of the lowest nonzero mode with negative chirality. In every sub-table, the first line lists the numbers of points selected and the value of chirality $c(n)$ on the iso-surfaces (listed in brackets); the lower lines list the coordinates of the centers of every lump and the numbers of points used (listed in brackets) for every lump.

The structures of the Zero mode (positive chirality):				
64 (3.50×10^{-4})	128 (2.84×10^{-4})	512 (1.48×10^{-4})	1024 (9.98×10^{-5})	2048 (6.61×10^{-5})
12.7 15.1 12.2 12.3 (64)	12.6 15.2 12.3 12.2 (128)	12.5 15.4 12.5 12.2 (512)	12.2 15.3 12.3 12.0 (1018)	12.5 15.4 12.5 12.2 (2018)
			4.6 0.5 7.0 7.3 (6)	4.8 0.5 6.7 7.5 (30)
The structures of the lowest nonzero mode with positive chirality:				
64 (1.39×10^{-4})	128 (1.06×10^{-4})	512 (6.74×10^{-5})	1024 (5.33×10^{-5})	2048 (4.16×10^{-5})
5.0 0.9 5.9 7.7 (64)	5.1 0.8 6.0 7.7 (128)	5.2 1.1 5.8 7.9 (304)	5.1 1.6 5.5 8.1 (496)	4.3 3.3 6.0 10.1 (1517)
	3.4 3.7 7.1 14.7 (7)	4.0 6.5 2.0 8.5 (4)	3.2 4.3 7.2 13.3 (283)	
		3.4 4.0 6.9 14.3 (101)	2.8 5.4 8.3 9.6 (9)	
	10.0 10.0 10.0 9.5 (2)	10.2 10.3 10.1 9.5 (57)	10.1 10.4 10.2 9.5 (139)	
		12.8 14.6 12.2 12.2 (37)	12.7 14.7 12.1 12.2 (106)	
				11.2 12.4 11.0 10.6 (531)
The structures of the lowest nonzero mode with negative chirality:				
64 (-2.10×10^{-4})	128 (-1.69×10^{-4})	512 (-9.23×10^{-5})	1024 (-6.77×10^{-5})	2048 (-4.87×10^{-5})
1.1 6.9 15.7 0.6 (64)	1.2 7.0 15.7 0.6 (128)	1.3 6.8 15.3 0.5 (512)	1.4 6.8 14.8 0.6 (1024)	1.7 6.6 15.2 0.6 (2048)

Then a comparison of the results obtained from overlap fermions and improved 5Li cooling is made. The improved

5Li cooling[7, 8] is a local minimization of the following 5Li lattice action

$$S = \sum_{i=1}^5 c_i S_{m_i, n_i}, \quad (1)$$

$$S_{m,n} = \frac{1}{m^2 n^2} \sum_{x, \mu, \nu} \text{Tr} \left[1 - \frac{1}{2} (R_{x, \mu\nu}^{m \times n} + R_{x, \mu\nu}^{n \times m}) \right], \quad (2)$$

where $R_{x, \mu\nu}^{m \times n}$ denotes the ordered product of SU(3) link matrices along $m \times n$ rectangles in the μ, ν plane, $(m_i, n_i) = (1,1), (2,2), (1,2), (1,3), (3,3)$ for $i = 1, \dots, 5$ and $c_1 = (19 - 55c_5)/9$, $c_2 = (1 - 64c_5)/9$, $c_3 = (-64 + 640c_5)/45$, $c_4 = 1/5 - 2c_5$, $c_5 = 1/20$. The general cooling with Wilson action has no stable instantons, even it existed in the gauge field, due to shrink and decay under cooling. But the 5Li cooling overcomes this difficulty to some extent. The 5Li action provides a small energy barrier against the decay of instantons of size $\rho \geq 2.3a$, thus preserves instantons over that size under arbitrary amounts of cooling[8]. After cooling, one can use topological charge density operator[2]

$$q(x) = -\frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{ReTr}[F_{\mu\nu}(x)F_{\rho\sigma}(x)] \quad (3)$$

to measure the topological charge distribution of the cooled configurations. Although this operator is inadequate for configurations of large fluctuations, it is adequate for the smooth configurations which emerge after several cooling steps. We found that the improved 5Li cooling and the overlap fermions (with APE smeared background gauge field) can generate similar structures if we adjust cooling steps. In Fig.2 we compared the 5Li cooling and overlap fermions results. Fig.2(a) is the chirality distribution of the lowest nonzero eigenmode of overlap-Dirac operator in a given time slice ($t = 9$) for a $16^3 \times 20$ lattice generated with the Iwasaki gauge action[10] at $\beta = 2.746$ (spacing $a = 0.08\text{fm}$). Fig.2(b) is the topological charge density distribution of cooled configuration with 15 5Li cooling steps for the same lattice. From Fig.2 one can see that 5Li cooling and overlap fermions filter out almost the same structure but the two methods differ from each other significantly, which suggests that the topological information thus obtained is physical. So these two methods can be combined to study the vacuum structure.

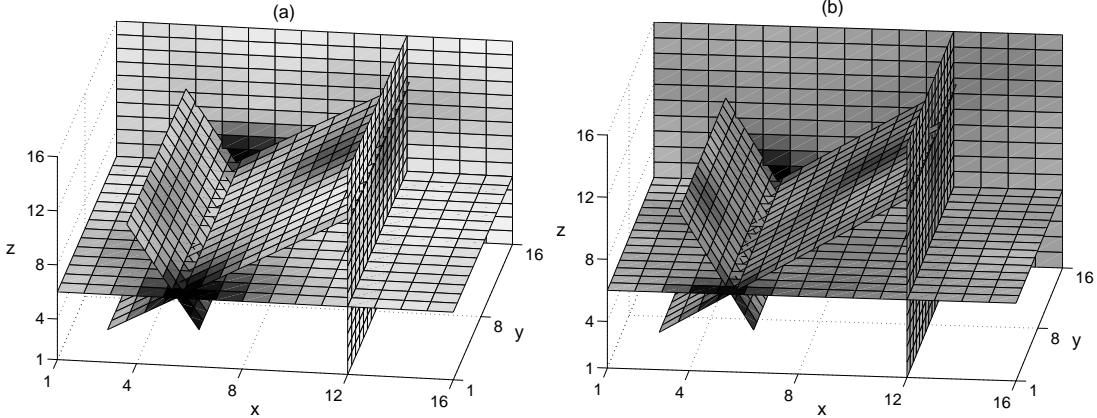


FIG. 2: The chirality $c(n)$ distribution of the lowest nonzero eigenmodes of overlap-Dirac operator (a) and the topological charge density distribution of 5Li cooled configuration(b) in a given time ($t = 9$) slice of a $16^3 \times 20$ lattice at $\beta = 2.746$ (spacing $a = 0.08\text{fm}$). Periodic boundary conditions are imposed. The darkness denotes high chirality and high topological charge density separately.

Moreover we found that if we take about 50 APE smearing steps all these three filter methods can produce similar results. In Fig.3 we showed the topological charge distribution of smoothed configuration (with 50 APE smearing steps) for the same lattice of Fig.2.

Now we can study further the topological charge of the structures of the vacuum. The topological charge Q of a structure within the range of radius r can be calculated by

$$Q(r) = \sum_{|x-x_0| \leq r} q(x), \quad (4)$$

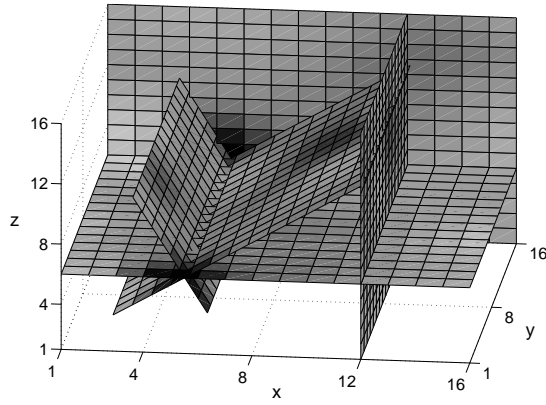


FIG. 3: The topological charge distribution of smoothed configuration (with 50 APE smearing steps) for the same lattice of Fig.2. The darkness denotes high topological charge density.

where x_0 is the center of the structure. If $|Q(r)|$ reaches 0.4, this structure may be regarded as an (anti-)instanton, and r can be regarded as its radius approximately, since the analytical instanton topological charge density is

$$Q_\rho(x) = \frac{6}{\pi^2 \rho^4} \left[\frac{\rho^2}{(x - x_0)^2 + \rho^2} \right]^4, \quad (5)$$

where ρ is the radius parameter, and the topological charge contribution within radius ρ of an instanton is 0.5. Sometimes two structures are close each other. If the two structures have the total topological charge which is smaller than one and greater than 0.4, they are considered as one structure; if their total topological charge greater than one the two structures are considered to be independent ones.

With above criterion, we get our results showed in Table 2. Our calculation is done on a $16^3 \times 20$ lattice with 8 gauge configurations generated with the Iwasaki gauge action[10] at $\beta = 2.746$ (spacing $a = 0.08\text{fm}$, the physical volume is 3.36fm^4) and cooled by improved 5Li method. From Table 2 we can see that for every configuration the total topological charge near an integer and is consistent with the number of zero modes, which are consistent with ILM. The mean density of structures is 1.23fm^{-4} and their mean radius is $\rho \approx 0.395\text{fm}$, which is a little bigger than those of ILM.

If we associate the overlap fermions with APE smearing, we can obtain the same result as above, i.e., there are the same number of structures for every configuration. The mean radius of instantons is $\rho \approx 0.397\text{fm}$. The total topological charge for every APE smeared configurations (with 50 APE steps) are listed in Table 2 also, which are close to the results of the improved 5Li cooled configurations. This confirms the result furthermore.

TABLE II: The number of zero modes, total topological charge and the number of structures for every configuration. For the total topological charge, the upper line calculated from 5Li cooled configurations and the lower line calculated from APE smeared configurations.

configuration	1	2	3	4	5	6	7	8
# of zero modes	1	3	1	1	0	3	1	1
total topological	-0.88	2.75	0.87	-0.95	0.06	2.76	-0.99	0.97
charge	-0.87	2.75	0.86	-0.97	-0.04	2.75	-1.01	0.99
# of structures	5	7	5	5	2	3	3	3

Our results show that even after APE smearing or 5Li cooling there are still short range fluctuations with very small topological charges. This confirms our suspicion that there are random fluctuation lump structures included in Horváth et al.'s results. If these random short wave length fluctuations are dropped, a dilute instanton liquid QCD vacuum structure appears. Based on these results we concluded that the dilute instanton liquid dominates the QCD vacuum. However, since the lattice volume is only 3.36fm^4 , the random fluctuations might be very serious. In addition we have to decrease the lattice spacing a to check if all of these results are stable against the lattice spacing changes.

Whether the large quantum fluctuation of the QCD vacuum conjectured by Witten[11] will destroy the instanton structure, our results show not, but needs to be studied further as well.

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- [1] E.V. Shuryak, Nucl. Phys. **B198**, 83 (1982); D.I. Diakonov and V.Y. Petrov, Nucl. Phys. **B245**, 259 (1984); T. Schäfer and E. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).
 - [2] M. C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, Phys. Rev. **D49**, 6039 (1994).
 - [3] M. Falcioni, M. Paciello, G. Parisi and B. Taglienti, Nucl. Phys. **B251**, 624 (1985); M. Albanese, et al. phys. Lett. **B192**, 163 (1987).
 - [4] I. Horváth, et al., Phys. Rev. **D66**, 034501 (2002).
 - [5] A. Hasenfratz and C. Nieter, Phys. Lett. **B439**, 366 (1998).
 - [6] T. DeGrand, A. Hasenfratz and T. Kovács, Nucl. Phys. **B520**, 301 (1998).
 - [7] Ph. de Forcrand, M.Garcia Perez and I.-O. Stamatescu, Nucl. Phys. **B** (Proc. Suppl.) **47**, 777 (1996); Ph. de Forcrand, M.Garcia Perez and I.-O. Stamatescu, Nucl. Phys. **B499**, 409 (1997).
 - [8] Ph. de Forcrand et al, Nucl. Phys. **B** (Proc. Suppl.) **63**, 549 (1998).
 - [9] H.Neuberger, Phys. Lett. **B417**, 141 (1998); **B427**, 353 (1998).
 - [10] Y. Iwasaki, Nucl. Phys. **B258**, 141 (1985); M. Okamoto et al, Phys. Rev. **D60**, 094510 (1999).
 - [11] E.Witten, Nucl. Phys. **B149**, 285 (1979); **B156**, 269 (1979).